

## Supporting Information for:

### **Assortative mating in an ecological context: Effects of mate choice errors and relative species abundance on the frequency and asymmetry of hybridization**

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## Appendix 1: Dependence of hybridization rate on relative species abundance

In this appendix, we will prove the following results:

- (1) The probability  $h_i$  that a female of species  $i$  mates with a heterospecific male is positively related to the propensity  $\beta_i$  to (wrongly) accept a heterospecific male and negatively related to the relative abundance  $p_i$  of species  $i$ .
- (2) The relative abundance  $C_i = p_i c_i$  of conspecific matings involving females of species  $i$  increases with the relative abundance  $p_i$  of this species.
- (3) The relative abundance  $H_i = p_i h_i$  of hybrid matings involving females of species  $i$  is a hump-shaped function of the relative abundance  $p_i$  of this species.
- (4) If the number of mating opportunities is unlimited ( $n \rightarrow \infty$ ), the maximum of  $H_i$  is attained at  $p_{i,\max} = \sqrt{\beta_i} / (\sqrt{\beta_i} + \sqrt{1 - \alpha_i})$ . The maximum is always attained at a value  $p_{i,\max} < 0.5$ , and any increase in the error rates  $\alpha_i$  or  $\beta_i$  leads to a shift of the maximum to the right.

These results are based on the assumption that the error rate  $\beta_i$  is constant. We also considered scenarios where  $\beta_i$  either increases or decreases with the relative abundance  $p_i$ . For the special case of unlimited mating opportunities ( $n \rightarrow \infty$ ), we can prove some analytical results for these scenarios. We will show:

- (5) The derivative of  $h_i$  is proportional to  $-\beta_i + p_i(1 - p_i)\beta_i'$ . As a consequence,  $h_i$  is not necessarily decreasing with  $p_i$  anymore.  $h_i$  may increase with  $p_i$ , but this can only happen if  $\beta_i$  is an increasing function ( $\beta_i' > 0$ ) and only at intermediate relative species abundance.
- (6) In comparison to the scenario of a constant error rate  $\beta_i$ ,  $H_i'$  changes by a term that has the same sign as  $\beta_i'$ . This implies that the maximum of  $H_i$  is attained at a larger value of  $p_i$  if  $\beta_i$  is an increasing function ( $\beta_i' > 0$ ), while it is attained at a smaller lower value of  $p_i$  if  $\beta_i$  is a decreasing function ( $\beta_i' < 0$ ).

Proof: For simplicity, we will drop the subscript  $i$  throughout the proof.

- (1) We will show that  $c = 1 - h$  increases with  $p$  and decreases with  $\beta$ . In the main text, we have derived that  $c$  is given by:

$$c = (1 - u) \cdot \frac{a}{a + b} + u \cdot p, \quad (1)$$

where  $a = p(1 - \alpha)$ ,  $b = (1 - p)\beta$  and  $u = (1 - a - b)^n$ .

- (1a) To investigate how  $c$  depends on  $p$ , we consider the derivative of  $c$  with respect to  $p$ . All derivatives with respect to  $p$  will be denoted by a prime. In case of  $c$ , this derivative is given by

$$c' = -u' \cdot \frac{a}{a + b} + (1 - u) \cdot \frac{a'(a + b) - a(a' + b')}{(a + b)^2} + u' \cdot p + u. \quad (2)$$

We have to show that  $c'$  is positive. Slightly rearranging (2) yields:

$$c' = u + (1-u) \cdot \frac{a'b - ab'}{(a+b)^2} + (-u') \cdot \left( \frac{a}{a+b} - p \right). \quad (3)$$

We will now show that all three terms on the right-hand side of (3) are positive, provided that  $\alpha$  and  $\beta$  are small enough to guarantee that  $1 - \alpha - \beta > 0$ . It is obvious that  $0 < u < 1$ . Hence, the first term on the rhs of (3) is positive. That the second term is positive follows from

$$a'b - ab' = (1 - \alpha) \cdot (1 - p) \beta - p(1 - \alpha) \cdot (-\beta) = (1 - \alpha) \beta > 0. \quad (4)$$

The third term on the rhs of (3) is also positive, since both factors are positive:

$$-u' = -n \cdot (1 - a - b)^{n-1} \cdot (-a' - b') = n \cdot u^{\frac{n-1}{n}} \cdot (1 - \alpha - \beta) > 0. \quad (5)$$

$$\frac{a}{a+b} - p = \frac{(1-p)a - pb}{a+b} = \frac{p(1-p)(1-\alpha) - p(1-p)\beta}{a+b} = \frac{p(1-p)(1-\alpha-\beta)}{a+b} > 0. \quad (6)$$

Hence  $c' > 0$ , implying that  $c$  increases with  $p$  while  $h$  decreases with  $p$ . Notice that inequality (6) implies that the probability of mating with a conspecific during the  $n$  mate choice rounds ( $a/(a+b)$ ) is larger than the probability of mating with a conspecific at random (which is equal to  $p$ ).

(1b) To show that  $c$  decreases with  $\beta$ , we have to show that  $\partial c / \partial \beta < 0$ , where  $\partial c / \partial \beta$  denotes the derivative of  $c$  with respect to  $\beta$ . The derivatives of  $a$ ,  $b$  and  $u$  with respect to  $\beta$  are given by  $\partial a / \partial \beta = 0$ ,  $\partial b / \partial \beta = 1 - p$  and  $\partial u / \partial \beta = -n \cdot (1 - p) \cdot u^{(n-1)/n}$ . Since  $\partial u / \partial \beta < 0$ , the derivative of the second term on the rhs of (1) with respect to  $\beta$  is negative. Hence,  $\partial c / \partial \beta < 0$  if we can show that the derivative of the first term on the rhs of (1) with respect to  $\beta$  is also negative. This derivative is given by

$$\frac{\partial}{\partial \beta} \left( (1-u) \cdot \frac{a}{a+b} \right) = n \cdot (1-p) \cdot u^{\frac{n-1}{n}} \cdot \frac{a}{a+b} - (1-u) \cdot \frac{a \cdot (1-p)}{(a+b)^2} = \frac{a \cdot (1-p)}{a+b} \cdot \left( n \cdot u^{\frac{n-1}{n}} - \frac{1-u}{a+b} \right). \quad (7)$$

We will show that the right-most term in brackets is negative. To see this, we make use of the relation  $a+b = 1 - u^{1/n}$ , which follows directly from the definition of  $u$ . We have to show that

$$n \cdot u^{\frac{n-1}{n}} - \frac{1-u}{1-u^{1/n}} < 0$$

or, equivalently, that

$$f(u) = 1 + (n-1) \cdot u - n \cdot u^{\frac{n-1}{n}} > 0 \quad (8)$$

for all  $u$  satisfying  $0 < u < 1$ . The derivative of  $f$ ,  $df/du = (n-1) \cdot (1 - u^{-1/n})$ , reveals that  $f$  is a decreasing function on the unit interval that takes on its minimal value  $f(1) = 0$  at  $u = 1$ . Hence,  $f(u) > 0$  for  $0 < u < 1$ , which completes the proof.

(2) The derivative of  $C = pc$  with respect to  $p$  is given by  $C' = c + pc'$ . Since  $c'$  is positive,  $C'$  is also positive. Hence  $C$  increases with  $p$ .

(3) In contrast,  $H = ph$  is not monotonic. A straightforward calculation shows that  $H(0) = H(1) = 0$  for the border cases  $p = 0$  and  $p = 1$ . The derivative of  $H$  is given by  $H' = h + ph' = 1 - c - pc'$ . Making use of (1) and (3), it is easy to see that  $H'(0) = 1$  and  $H'(1) = -c'(1) < 0$ . In other words,  $H$  increases with  $p$  at a small species abundance  $p \approx 0$ , while it decreases with  $p$  at  $p \approx 1$ . Hence,  $H$  has at least one maximum at an intermediate abundance  $0 < p < 1$ . To prove that  $H$  is a hump-shaped function of  $p$ , we will now prove that this maximum is unique. To see this, notice that the equation  $H' = h + ph' = 0$  is equivalent to  $h'/h = -1/p$ . It is obvious that the rhs of this equation is an increasing function of  $p$ . In contrast, the lhs of this equation is a decreasing function of  $p$ :  $h'/h$  is the derivative of  $\ln(h)$ , which is decreasing with  $p$  since (as shown above)  $h$  is a decreasing function of  $p$ . Since a decreasing and an increasing function intersect at most once, the equation  $H' = h + ph' = 0$  has a unique solution.

(4) If the number of mating opportunities is unlimited ( $n \rightarrow \infty$ ),  $u$  approaches 1. Accordingly,  $c$  and  $h$  simplify to:  $c = a/(a+b)$  and  $h = b/(a+b)$ . Accordingly, the derivatives of  $h$  and  $H = ph$  are of the form

$$h' = \frac{b'a - ba'}{(a+b)^2} = -\frac{\beta(1-\alpha)}{(a+b)^2}, \quad (9)$$

$$H' = h + ph' = \frac{b(a+b) + p(b'a - ba')}{(a+b)^2} = \frac{\beta \cdot ((1-p)^2 \beta - p^2(1-\alpha))}{(a+b)^2}. \quad (10)$$

Therefore the maximum of  $H$  is characterized by  $p^2(1-\alpha) = (1-p)^2 \beta$ , yielding

$$\frac{p_{\max}}{1-p_{\max}} = \sqrt{\frac{\beta}{1-\alpha}} \Leftrightarrow p_{\max} = \frac{\sqrt{\beta}}{\sqrt{\beta} + \sqrt{1-\alpha}}. \quad (11)$$

Hence, the position of the maximum ( $p_{\max}$ ) is positively related to both  $\alpha$  and  $\beta$ . Moreover, our assumption  $1-\alpha-\beta > 0$  implies  $\sqrt{1-\alpha} > \sqrt{\beta}$  and, hence,  $p_{\max} < 0.5$ .

(5) Let us now assume that  $\beta$  is not constant, but a function of  $p$ . As a consequence, eqn (9) changes to

$$h' = \frac{b'a - ba'}{(a+b)^2} = \frac{(1-\alpha) \cdot (-\beta + p(1-p)\beta')}{(a+b)^2}. \quad (12)$$

This is negative if  $\beta' \leq 0$ , but if  $\beta$  is an increasing function,  $h$  may increase (at intermediate values of  $p$ ) if  $\beta'$  is sufficiently large. More precisely:

$$h' > 0 \Leftrightarrow \beta'/\beta = \frac{d}{dp} \ln(\beta) > \frac{1}{p(1-p)}. \quad (13)$$

(6) If  $\beta$  is a function of  $p$ , eqn (8) changes to

$$H' = h + ph' = \frac{\beta \cdot ((1-p)^2 \beta - p^2(1-\alpha)) + \beta' p^2(1-p)}{(a+b)^2}. \quad (14)$$

Now the condition for a maximum ( $H' = 0$ ) does not have a straightforward solution anymore. We can, however, ask, how the location of the maximum is affected if  $\beta$  is made dependent on  $p$ . This depends on the term  $\beta' p^2 (1-p)$  on the rhs of eqn (14), which has the same sign as  $\beta'$ . If  $\beta' > 0$ ,  $H$  is still increasing at the former maximum  $p_{\max}$ ; in other words, the maximum is shifted to the right. Conversely, the maximum is shifted to the left if  $\beta' < 0$ .

## 2. Relative species abundance and sex-ratios

The hybridization model (page 7-9, main manuscript) does not consider variation in sex-ratios.

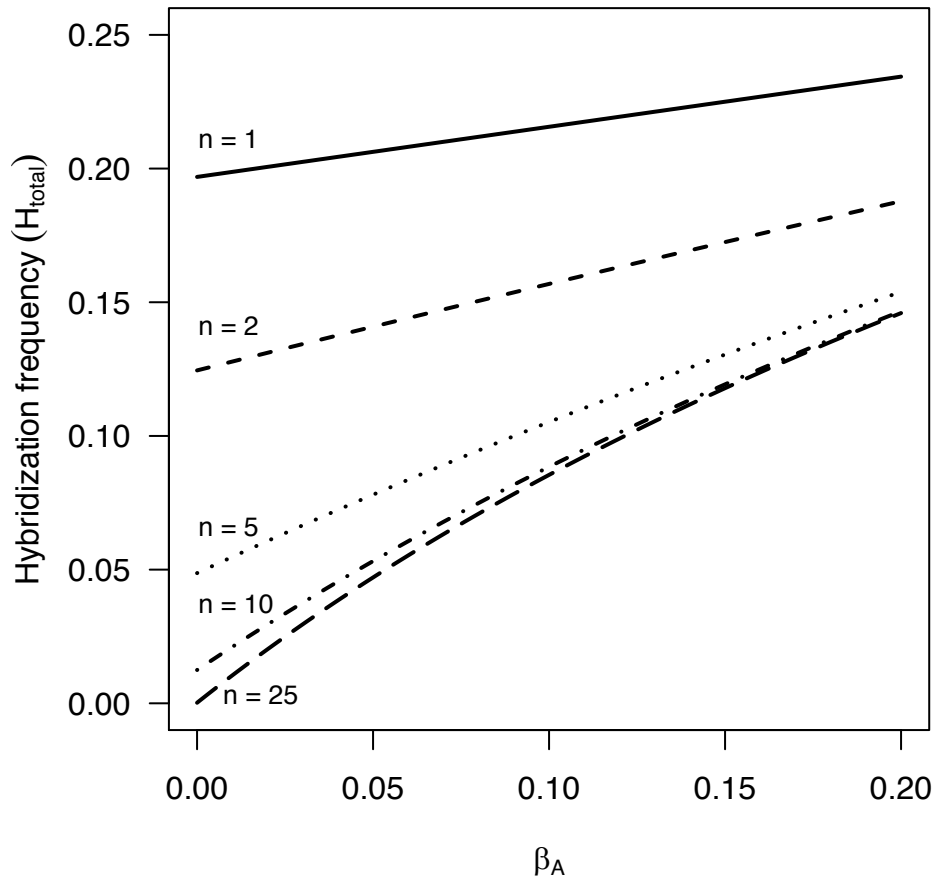
Our parameter  $p_i$ , the relative abundance of species  $i$ , refers to the relative abundance of the males of species  $i$ . If the sex ratio is the same in both species, the relative abundance of males corresponds to the relative abundance of all individuals. If the sex ratios (proportion males)  $s_A$  and  $s_B$  are not the same, the definitions of  $p_A$  and  $p_B$  have to be changed to:

$$p_A = s_A * n_A / (s_A * n_A + s_B * n_B),$$

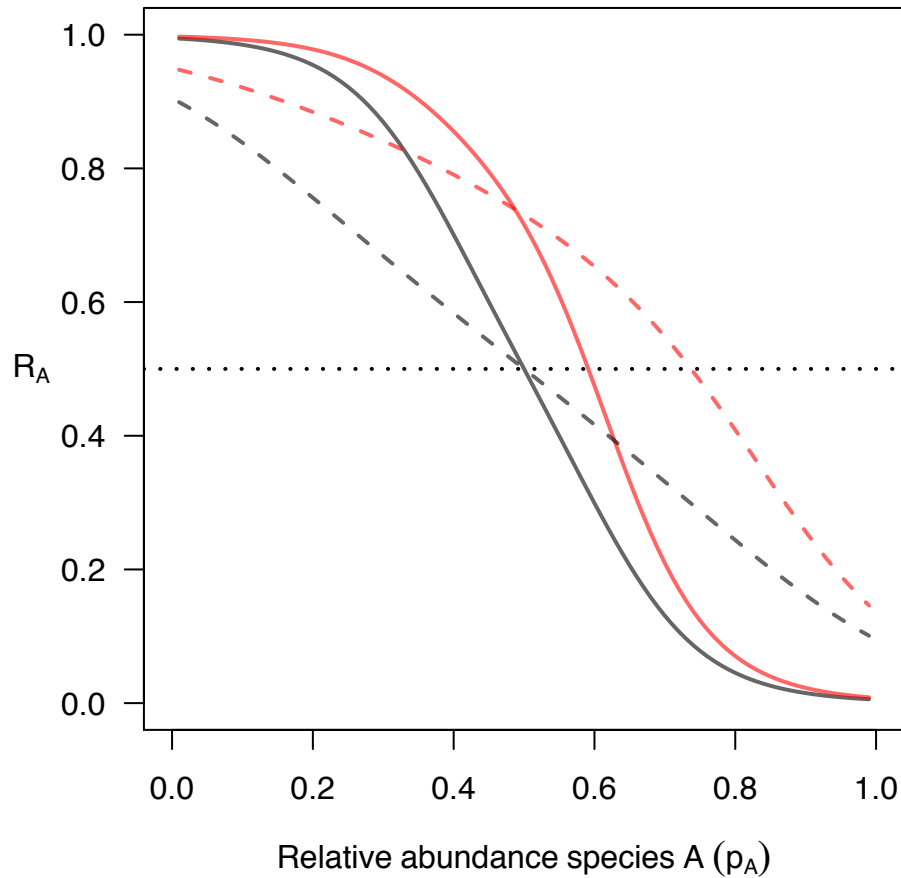
and similarly for  $p_B$ . As before,  $n_A$  and  $n_B$  are the species abundances.

## 2. Supporting figures

Supporting Figure 1. Dependence of the maximal hybridization frequency (the maximal value of  $H_{\text{total}}$ ) on the error rate  $\beta_i$  ( $\beta_A = \beta_B$ ) for various values of  $n$ , the number of males sampled during mate choice. In all cases  $\alpha_A = \alpha_B = 0.05$  and  $p = 0.25$ .



Supporting Figure 2. Relative contribution  $R_A = H_A/H_{\text{total}}$  of females of species A to population-level hybridization as a function of the relative abundance  $p_A$  of species A for the four scenarios considered in Figure 3. The line type reflects whether  $\beta_i$ -values are small (solid lines;  $\bar{\beta}_i = 0.005$ ) or large (dashed lines;  $\bar{\beta}_i = 0.1$ ) and the colour reflects whether  $\beta_i$ -values are symmetric (black lines;  $\beta_A = \beta_B$ ) or asymmetric (red lines;  $\beta_A \neq \beta_B$ ).





Supporting Figure 3. Comparing three scenarios for the relationship between  $\beta_A$  and relative species abundance of species A, where  $\beta_A$  is constant (0.25; grey), linearly increasing (blue) or decreasing (red) between 0.1 and 0.4 (A)  $\beta_B = 0.25 = \text{constant}$  and  $\alpha_A = \alpha_B = 0.05$ . For all three scenarios, panel B shows how  $R_A$ , the proportion hybridization events involving a female of species A, is changing with the relative abundance of species A.

